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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION (UPDATED)

CLASS - 12 (PCM)
Question Paper Code : UN489

KEY

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SOLUTIONS

MATHEMATICS

01. (D) $P = \{(a, b): \sec^2 a - \tan^2 b = 1\}$

For reflexive :

$\sec^2 a - \tan^2 a = 1$ (true $\forall$ a)

For symmetric :

$\sec^2 b - \tan^2 a = 1$

LHS

$1 + \tan^2 b - (\sec^2 a - 1)$

$= 1 + \tan^2 b - \sec^2 a + 1$

$= -(\sec^2 a - \tan^2 b) + 2$

$= -1 + 2 = 1$

So, Relation is symmetric

For transitive :

if $\sec^2 a - \tan^2 b = 1$ and

$\sec^2 b - \tan^2 c = 1$

$\sec^2 a - \tan^2 c = (1 + \tan^2 b) - (\sec^2 b - 1)$

$= -\sec^2 b + \tan^2 b + 2$

$= -1 + 2 = 1$

So, Relation is transitive

Hence, Relation P is an equivalence relation

02. (B) $-\sin^{-1}\left(\frac{12}{3}\right)-\sin^{-1}\left(\frac{3}{5}\right)$

$$\sin^{-1}\left(\frac{12}{3}\times\frac{4}{5}-\frac{5}{13}\times\frac{3}{5}\right)$$

$$=\sin^{-1}\left(\frac{48-15}{65}\right)$$

$$=\sin^{-1}\left(\frac{33}{65}\right)$$

$$\left[\because \sin^{-1}x-\sin^{-1}y=\sin^{-1}\left\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\right\}\right]$$

$$=\cos^{-1}\left(\frac{56}{65}\right)=\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$$

03. (C) $A=\begin{pmatrix}0 & 2q & r\\ p & q & -r\\ p & -q & r\end{pmatrix}$

$$\therefore AA^T=\begin{pmatrix}0 & 2q & r\\ p & q & -r\\ p & -q & r\end{pmatrix}\times\begin{pmatrix}0 & p & p\\ 2q & q & -q\\ r & -r & r\end{pmatrix}$$

$$=\begin{pmatrix}4q^2+r^2 & 2q^2-r^2 & -2q^2+r^2\\ 2q^2-r^2 & p^2+q^2+r^2 & p^2-q^2-r^2\\ -2q^2+r^2 & p^2-q^2-r^2 & p^2+q^2+r^2\end{pmatrix}$$

Given, $AA^T=I$

$\therefore 4q^2+r^2=p^2+q^2+r^2=1$

$\Rightarrow p^2-3q^2=0$ and $r^2=1-4q^2$

and $2q^2-r^2=0\Rightarrow r^2=2q^2$

$\therefore p^2=\frac{1}{2}$, $q^2=\frac{1}{6}$ and $r^2=\frac{1}{3}$

$\therefore |p|=\frac{1}{\sqrt{2}}$

04. (A) $|A|=\begin{vmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{vmatrix}$

$=2(2b^2+2-b^2)-b(2b-b)+1(b^2-b^2-1)$

$=2b^2+4-b^2-1=b^2+3$

$$\frac{|A|}{b}=b+\frac{3}{b}$$

$$\because \frac{b+\frac{3}{b}}{2}\ge\left(b\times\frac{3}{b}\right)^{\frac{1}{2}}\Rightarrow b+\frac{3}{b}\ge 2\sqrt{3}$$

$$\therefore \frac{|A|}{b}\ge 2\sqrt{3}$$

Minimum value $\frac{|A|}{b}$ of is $2\sqrt{3}$

05. (B) $\Delta=\frac{1}{2}\begin{vmatrix}0 & 2 & 1\\ 1 & -1 & 1\\ x' & y' & 1\end{vmatrix}=5$

$\Rightarrow -2(1-x')+(y'+x')=\pm 10$

$\Rightarrow -2+2x'+y'+x'=\pm 10$

$\Rightarrow 3x+y'=12$ or $3x'+y'=-8$

$\therefore \lambda=3,-2$

06. (A) If the function is continuous at $x=0$, then

$\lim_{x\to 0} f(x)$ will exist and

$f(0)=\lim_{x\to 0} f(x)$

Now, $\lim_{x\to 0} f(x)=\lim_{x\to 0}\left(\frac{1}{x}-\frac{k-1}{e^{2x}-1}\right)$

$$=\lim_{x\to 0}\left(\frac{e^{2x}-1-kx+x}{(x)\left(e^{2x}-1\right)}\right)$$

$$= \lim_{x\to 0}\left[\frac{\left(1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{2!}\right)-1-kx+x}{(x)\left(\left(1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+....\right)-1\right)}\right]$$

$$= \lim_{x\to 0}\left[\frac{(3-k)x+\frac{4x^2}{2!}+\frac{8x^3}{3!}+....}{\left(2x^2+\frac{4x^3}{2!}+\frac{8x^3}{3!}+....\right)}\right]$$

For the limit to exist, power of x in the numerator should be greater than or equal to the power of x in the denominator. Therefore, coefficient of x in numerator is equal to zero

$\Rightarrow 3 - k = 0$

$\Rightarrow k = 3$

So the limit reduces to

$$\lim_{x\to 0}\frac{(x^2)\left(\frac{4}{2!}+\frac{8x}{3!}+....\right)}{(x^2)\left(2+\frac{4x}{2!}+\frac{8x^2}{3!}+....\right)}$$

$$\lim_{x\to 0}\frac{\frac{4}{2!}+\frac{8x}{3!}+....}{2+\frac{4x}{2!}+\frac{8x^2}{3!}+....}$$

Hence, $f(0) = 1$

07. (B) Let $f : R \to R$, with $f(0) = f(1) = 0$ and $f'(0) = 0$

$\because$ $f(x)$ is differentiable and continuous and

$f(0) - f(1) = 0$

Then by Rolle's theorem,

$f'(c) = 0,\ c \in (0, 1)$

Now again

$\because\ f'(c) = 0,\ ,\ f'(0) = 0$

Then, again by Rolle's theorem,

$f''(x) = 0$ for some $x \in (0, 1)$

08. (A) $$f(x)=\frac{x}{\sqrt{a^2+x^2}}-\frac{(d-x)}{\sqrt{b^2+(d-x)^2}}$$

$$=\frac{x}{\sqrt{a^2+x^2}}+\frac{(x-d)}{\sqrt{b^2+(x-d)^2}}$$

$$f'(x)=\frac{\sqrt{a^2+x^2}-\frac{x(2x)}{2\sqrt{a^2+x^2}}}{(a^2+x^2)}$$

$$+\frac{\sqrt{b^2+(x-d)^2}-\frac{(x-d)2(x-d)}{2\sqrt{b^2+(x-d)^2}}}{\left(b^2+(x-d)^2\right)}$$

$$=\frac{a^2+x^2-x^2}{(a^2+x^2)^{\frac{3}{2}}}+\frac{b^2+(x-d)^2-(x-d)^2}{\left(b^2+(x-d)^2\right)^{\frac{3}{2}}}$$

$$=\frac{a^2}{(a^2+x^2)^{\frac{3}{2}}}+\frac{b^2}{\left(b^2+(x-d)^2\right)^{\frac{3}{2}}}>0$$

$\Rightarrow f'(x) > 0,\ x \in R$

$\Rightarrow$ $f(x)$ is increasing function

Hence, $f(x)$ is increasing function

09. (D) Let $$I=\int\frac{\cos x\,dx}{\sin^3 x(1+\sin^6 x)^{\frac{2}{3}}}$$

$$=f(x)\left(1+\sin^6 x\right)^{\frac{1}{\lambda}}+c$$

If $\sin x = t$

then, $\cos x\,dx = dt$

$$I=\int\frac{dt}{t^3\left(1+t^6\right)^{\frac{2}{3}}}=\int\frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$$

Put $1+\frac{1}{t^6}=r^3 \Rightarrow \frac{dt}{t^7}=\frac{-1}{2}r^2dr$

$$\therefore \quad I=-\frac{1}{2}\int\frac{r^2dr}{r^2}=-\frac{1}{2}r+c$$

$$=-\frac{1}{2}\left(\frac{\sin^6 x+1}{\sin^6 x}\right)^{\frac{1}{3}}+c$$

$$=-\frac{1}{2\sin^2 x}\left(1+\sin^6 x\right)^{\frac{1}{3}}+c$$

$f(x)=-\frac{1}{2}\operatorname{cosec}^2 x$ and $\lambda = 3$

[from equ (i)]

$$\therefore \lambda f\left(\frac{\pi}{3}\right)=-2$$

10. (A) $$\frac{k}{6}=\int_0^{\frac{1}{2}}\frac{x^2}{\left(1-x^2\right)^{\frac{3}{2}}}dx$$

Let $x = \sin\theta$; $dx = \cos\theta \, d\theta$

then $$\int_0^{\frac{1}{2}}\frac{x^2}{\left(1-x^2\right)^{\frac{3}{2}}}dx=\int_0^{\frac{\pi}{6}}\frac{\sin^2\theta\cos\theta}{\cos^3\theta}d\theta$$

$$\therefore \frac{k}{6}=\int_0^{\frac{\pi}{6}}\frac{\sin^2\theta}{\cos^3\theta}\cos\theta d\theta$$

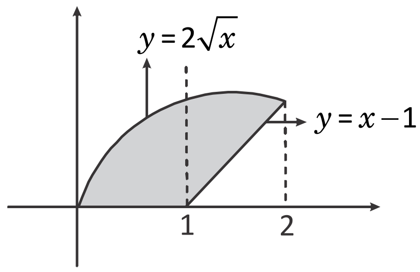
$$\Rightarrow \frac{k}{6}=\int_0^{\frac{\pi}{6}}\tan^2\theta\, d\theta=\int_0^{\frac{\pi}{6}}\left(\sec^2\theta-1\right)d\theta$$

$$\Rightarrow \frac{k}{6}=\left(\tan\theta-\theta\right)_0^{\frac{\pi}{6}}$$

$$=\left(\frac{1}{\sqrt{3}}-\frac{\pi}{6}\right)=\frac{2\sqrt{3}-x}{6}$$

$$\Rightarrow k=2\sqrt{3}-\pi$$

11. (A) $[x] = 0$ when $x \in [0, 1)$ and $[x] = 1$ when $x \in [1, 2)$



$$y=\begin{cases}0 & 0\le x<1\\ x-1 & 1\le x<2\end{cases}$$

$$\therefore \ A=\int_0^2 2\sqrt{x}dx-\frac{1}{2}(1)(1)$$

$$=\left[\frac{4x^{\frac{3}{2}}}{3}\right]_0^2-\frac{1}{2}=\frac{8\sqrt{2}}{3}-\frac{1}{2}$$

12. (A) Since, $x^2 = 4b(y + b)$

$x^2 = 4by + 4b^2$

$2x = 4by'$

$$\Rightarrow b=\frac{x}{2y'}$$

So, differential equation is

$$x^2=\frac{2x}{y'}y+\left(\frac{x}{y'}\right)^2$$

$x(y')^2 = 2yy' + x$

13. (A)

$$a\cos\theta=b\cos\left(\theta+\frac{2\pi}{3}\right)=c\cos\left(\theta+\frac{4\pi}{3}\right)=k$$

$$a=\frac{k}{\cos\theta},b=\frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)},c=\frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$$

ab + bc + ca

$$=k^2\frac{\left[\cos\left(\theta+\frac{4\pi}{3}\right)+\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)\right]}{\cos\left(\theta+\frac{4\pi}{3}\right)\times\cos\theta\times\cos\left(\theta+\frac{2\pi}{3}\right)}$$

$$=k^2\left[\frac{\cos\theta+2\cos(\theta+\pi)\cos\left(\frac{\pi}{3}\right)}{\cos\theta\times\cos\left(\theta+\frac{2\pi}{3}\right)\times\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

$$=k^2\left[\frac{\cos\theta-2\cos\theta\times\frac{1}{2}}{\cos\theta\times\cos\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta+\frac{4\pi}{3}\right)}\right]=0$$

$$\cos\phi=\frac{\left(a\hat{i}+b\hat{j}+c\hat{k}\right)\left(b\hat{i}+c\hat{j}+a\hat{k}\right)}{\sqrt{a^2+b^2+c^2}\sqrt{b^2+c^2+a^2}}$$

$= ab + bc + ca = 0$

$$\phi=\frac{\pi}{2}$$

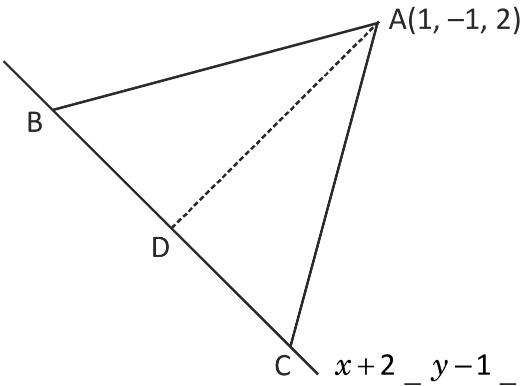
14. (D) Let a point D on $BC = (3\lambda - 2, 1, 4\lambda)$ $\overline{AD}$

$= (3\lambda - 3)\hat{i} + 2\hat{j} + (4\lambda - 2)\hat{k}$

$\because$ $\overline{AD} \perp \overline{BC}$, $\therefore$ $\overline{AD}\cdot\overline{BC} = 0$

$\Rightarrow$ $(3\lambda - 3) + 3 + 2(0) + (4\lambda - 2)4 = 0$

$$\Rightarrow \lambda = \frac{17}{25}$$



$$\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$$

Hence, $D = \left(\frac{1}{25}, 1, \frac{68}{25}\right)$

$$|AD|=\sqrt{\left(\frac{1}{25}-1\right)^2+(2)^2+\left(\frac{68}{25}-2\right)^2}$$

$$=\sqrt{\frac{(24)^2+4(25)^2+(18)^2}{25}}=\sqrt{\frac{3400}{25}}=\frac{2\sqrt{34}}{5}$$

Area of triangle $= \frac{1}{2}\times|BC| \times |AD|$

$$=\frac{1}{2}\times5\times\frac{2\sqrt{34}}{5}=\sqrt{34} \quad [\mapsto \ BC = 5]$$

15. (D) Probability of sum getting 6, $P(A)=\frac{5}{36}$

Probability of sum getting 7,

$$P(B)=\frac{6}{36}=\frac{1}{6}$$

$P(A \text{ wins}) = P(A)+P(\overline{A})P(\overline{B})P(A)$

$+P(\overline{A})\times P(\overline{B})P(\overline{A})P(\overline{B})P(A)+....$

$$\Rightarrow \frac{5}{36}+\left(\frac{31}{36}\right)\left(\frac{30}{36}\right)\left(\frac{5}{36}\right)+....\infty$$

$$\Rightarrow \frac{5}{36}\left(1+\frac{155}{216}+\left(\frac{155}{216}\right)^2+....\infty\right)$$

$$\Rightarrow \frac{\frac{5}{36}}{\frac{61}{216}}=\frac{30}{61} \qquad \left(\because S_\infty=\frac{a}{1-r}\right)$$

16. (B) Since $x<y, y<z \Rightarrow x<z \ \forall \ x,y,x \in$ N

$\therefore$ x R y, y R z $\Rightarrow x$ R z

$\therefore$ relation is transitive

$\because$ $x<y$ does not give $y<x$

$\therefore$ relation is not symmetric

$\because$ $x<x$ does not hold

$\therefore$ relation is not reflexive

17. (D) Number of ways of selecting 3 elements in A such that f(x) = y_2 is ${}^7C_3$ is . Now for remaining 4 elements in A, we have 2 elements in B

Total number of onto functions

$$= {}^7C_3\times(2^4-2)={}^7C_3\times14$$

18. (B) $f(x) = ax + b$ is an onto function from $[-1, 1]$ to $[0, 2]$

$\Rightarrow f(-1) = 0, f(1) = 2$ or $f(-1) = 2, f(1) = 0$

$\Rightarrow -a + b = 0, a + b = 2$ or

$-a + b = 2, a + b = 0$

$\Rightarrow a = 1, b = 1$ or $a = -1, b = 1$

$a > 0 \Rightarrow f(x) = x + 1$

$$\cot\left[\text{Tan}^{-1}\frac{1}{7}+\text{Tan}^{-1}\frac{1}{8}+\text{Tan}^{-1}\frac{1}{5}\right]$$

$$=\cot\text{Tan}^{-1}\left[\frac{\frac{1}{7}+\frac{1}{8}+\frac{1}{5}-\frac{1}{7}\times\frac{1}{8}\times\frac{1}{5}}{1-\frac{1}{7}\times\frac{1}{8}-\frac{1}{8}\times\frac{1}{5}-\frac{1}{5}\times\frac{1}{7}}\right]$$

$$=\cot\text{Tan}^{-1}\left[\frac{40+35+56-1}{280-5-7-8}\right]$$

$$=\cot\text{Tan}^{-1}\left(\frac{130}{260}\right)=2=f(1)$$

19. (C) If A is the required even and S is the sample space then $n(S) = 6^3 = 216$

$n(A) =$ Coeff. of x^k in

$(x + x^2 + x^3 + x^4 + x^5 + x^6)^3$

$$=\text{Coeff. of } x^k \text{ in }\left[\frac{x\left(1-x^6\right)}{1-x}\right]^3$$

$=$ Coeff. of x^k in $x^3(1 - x^6)^3(1 - x)^{-3}$

$=$ Coeff. of x^{k-3} in

$$\left(1-3x^6+3x^{12}-x^{18}\right)\left(1+{}^3C_1x+{}^4C_2x^2+....\right)$$

$9\le k\le 14\Rightarrow 6\le k-3\le 11$

$\Rightarrow n(A) =$ Coeff. of x^{k-3} is

$^{(k-1)}C_{k-3}-3^{(k-7)}C_{k-9} = {}^{(k-1)}C_2-3^{(k-7)}C_2$

$$=\frac{(k-1)(k-2)}{2}-\frac{3(k-7)(k-8)}{2}$$

$$=\frac{k^2-3k+2-3\left(k^2-15k+56\right)}{2}$$

$$=\frac{-2k^2+42k-166}{2}$$

$= 21k - k^2 - 83$

$$\therefore P(A)=\frac{n(A)}{n(S)}=\frac{21k-k^2-83}{216}$$

20. (D) a, b, c are coplanar $\Rightarrow \begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}\neq 0$

$$\begin{vmatrix}a & a^2 & 1+a^3\\ b & b^2 & 1+b^3\\ c & c^2 & 1+c^3\end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix}a & a^2 & 1\\ b & b^2 & 1\\ c & c^2 & 1\end{vmatrix}+\begin{vmatrix}a & a^2 & a^3\\ b & b^2 & b^3\\ c & c^2 & c^3\end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}+abc\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}=0$$

$\Rightarrow 1 + abc = 0$

$\Rightarrow abc = -1$

21. (C) We have $l=\frac{1}{\sqrt{2}}, m=-\frac{1}{2}$

As $l^2 + m^2 + n^2 = 1$

we have $n^2=\frac{1}{4}\Rightarrow n=\frac{1}{2}$

We take positive values, so

$n=\frac{1}{2}\Rightarrow\cos\theta=\frac{1}{2}$

$\therefore \theta = 60°$

22. (A) $\frac{dy}{dx}+2y=f(x)$ is a linear equation in y

I.F = $e^{\int 2dx} = e^{2x}$

$$\therefore ye^{2x} = \int f(x)e^{2x}dx = \frac{e^{2x}}{2}+c \text{ if } x\in[0,1]$$

$$y(0) = 0 \Rightarrow \frac{1}{2} + c = 0 \Rightarrow c = -\frac{1}{2}$$

$$\therefore ye^{2x} = \frac{e^{2x}}{2}-\frac{1}{2} \Rightarrow y = \frac{1}{2}-\frac{1}{2}e^{-2x}$$

$$\text{Now } y\left(\frac{3}{2}\right) = \frac{1}{2}-\frac{1}{2}e^{-3} = \frac{e^3-1}{2e^3}$$

23. (D) $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}\,dx$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi-x}{1+\sin(\pi-x)}\,dx = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi-x}{1+\sin x}\,dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi}{1+\sin x}\,dx - I$$

$$\Rightarrow 2I = \pi\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}\,dx$$

$$= \pi\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\sec^2 x-\sec x\tan x\right)\,dx$$

$$= \pi\left[\tan x-\sec x\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \pi\left[-1+\sqrt{2}-1+\sqrt{2}\right] = 2\pi\left(\sqrt{2}-1\right)$$

$$\Rightarrow I = \pi\left(\sqrt{2}-1\right)$$

24. (B)

$$f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+....+\frac{(-1)^n f^{(n)}(1)}{n!}=$$

$$=1-\frac{n}{1!}+\frac{n(n-1)}{2!}-\frac{n(n-1)(n-2)}{3!}+....+(-1)^n(1)$$

$$= (1-1)^n = 0$$

25. (A) If r is the radius of the spherical balloon 49 minutes after the leakage began then

$$4500\,\pi - \frac{4}{3}\,\pi\, r^3 = 72\pi\,(49) \Rightarrow \frac{4}{3}\,\pi\, r^3$$

$$= 4500\,\pi - 3528\pi \Rightarrow \frac{4}{3}\, r^3 = 972$$

$$\Rightarrow r^3 = 729 \Rightarrow r = 9$$

$$V = \frac{4}{3}\pi\, r^3$$

$$\Rightarrow \frac{dV}{dt} = 4\pi\, r^2\,\frac{dr}{dt}$$

$$\Rightarrow 72\pi = 4\pi\,(9)^2\,\frac{dr}{dt} = \frac{dr}{dt} = \frac{2}{9}$$

PHYSICS

26. (D) Current through arm AE = 2 + 1 = 3 A

Current through arm ED = Current through arm AE = 3 A

Current through arm DB = 3 – 2 = 1 A

Let, V_A and V_B be the electric potential at A and B

Then, $V_A - V_E = 1 \times 3 = 3$ V

$V_E - V_D = 3 \times 3 = 9$ V

$V_D - V_B = 3 \times 1 = 3$ V

$\therefore$ $V_A - V_B = (V_A - V_E) + (V_E - V_D) + (V_D - V_B)$

= 3 + 9 + 3 = 15 V.

27. (A) $\frac{n_p}{n_s}=\frac{5}{4}$ $\quad E_p = 220$ V

As $\frac{I_p}{I_s}=\frac{n_s}{n_p}=\frac{4}{5}$

28. (D) This is because copper is a diamagnetic material

29. (C) Dipole moment = p = charge × separation

$= q \times 2l = 1.602 \times 10^{-19} \times 4 \times 10^{-10}$

$= 6.408 \times 10^{-29}$ C m

Torque = pE sin θ

$= 6.408 \times 10^{-29} \times 3 \times 10^{5} \times \sin 30°$

$= 9.612 \times 10^{-24}$ N m

30. (D) In the first case, energy emitted,

$E_1 = 2E - E = E$

In the second case, energy emitted,

$$E_2 = \frac{4E}{3} - E = \frac{E}{3}$$

As E_2 is $\frac{1}{3}$rd, λ_2 must be 3 times, i.e., 3λ

31. (C) $$I_g = \frac{10 \times 10^{-3}}{5} = 2 \times 10^{-3} \text{ A}$$

$$R = \frac{V}{I_g} - G = \frac{1}{2 \times 10^{-3}} - 5 = 495\,\Omega$$

32. (B) From the condition of no emergence

$$\mu = \frac{1}{\sin\frac{A}{2}}$$

Here, A = 90°, therefore

$$\mu > \frac{1}{\sin 45°} \text{ or } \mu > \sqrt{2}$$

In this case as $\mu = \frac{3}{2} > \sqrt{2}$, therefore, light ray will not emerge out for any angle of incidence.

33. (C) Mass per nucleon in a hydrogen atom is slightly greater than mass per nucleon in oxygen because in the latter, some mass is appearing as binding energy.

34. (D) For diffraction of circular aperture. The condition of 1st minima is

$$d\sin\theta_1 = (1)\lambda$$

$$5 \cdot \left(\frac{x}{f}\right) \simeq (1)\lambda$$

$$\frac{5 \times x}{100} = 5 \times 10^{-5}$$

$x = 10^{-3}$ cm

35. (D) Three plates in a parallel plate capacitor, gives rise to two capacitors. If there are N plates, then there will be (N – 1) capacitors.

Given, N = 201 plates

Dielectric constant = ε_r = 2.5

Separation between the plates = d = 0.001 cm = 10^{-5} m

Area = 15 × 30 = 450 cm^2

$= 4.50 \times 10^{-2}$ m^2

$$\text{Capacitance} = C = \frac{\varepsilon_0 \varepsilon_r A}{d} \times (N-1)$$

$$= \frac{8.85 \times 10^{-12} \times 4.50 \times 10^{-2} \times 2.5 \times 200}{10^{-5}}$$

$= 19.91 \times 10^{-6}$ F

36. (B) X-rays are produced when there is a vacancy for the electron on inner complete orbits of an atom and jumping of electrons takes place from higher orbit to lower energy orbit of atom.

37. (C) $$\frac{1}{2} m v^2 = \frac{hc}{\lambda} - \phi_0 (\text{in eV})$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{4000 \times 10^{-10} \times 1.6 \times 10^{-19}} - 2$$

$= 3.1 - 2 = 1.1$ eV $= 1.1 \times 1.6 \times 10^{-19}$ J

$$v = \left[\frac{2 \times 1.1 \times 1.6 \times 10^{19}}{(9.1 \times 10^{-31})}\right]^{1/2}$$

$= 6.2 \times 10^{5}$ m/s.

38. (C) For a long solenoid, $B = \mu_0 nI$.

Where n is the no. of turns per unit length.

When length of solenoid is doubled, the no. of turns per unit length will remain unchanged.

$$B' = \frac{\mu_0 nI}{2L2} = \frac{B}{4}$$

39. (C) At any point over the spherical Gaussian surface, net electric field is vector sum of electric fields due to $+q_1' - q_1$ and q_2.

40. (B) As $e = M\frac{dI}{dt}$

$\therefore$ $M = \frac{e}{dI/dt} = \frac{2\times10^{-3}}{15.0} = 1.67\times10^{-3}$ H

As $\phi = MI$

$\therefore$ $\phi = 1.67\times10^{-3}\times3.6 = 6\times10^{-3}\,\text{Wb} = 6\,\text{m Wb}$

CHEMISTRY

41. (D) Hint: The deposition of K means the reduction of K+ ions to K metal on cathodes. This can be represented by the following reduction half-reaction.

$K^+ + e^- \longrightarrow K$

IF 39 g = 1 × 96500 C = 96500 C

This equation shows that :

39g of K are deposited by 96500 C

$\therefore$ 19.5g of K are deposited by

$\frac{96500\times19.5}{39}C = 48250\,C$

The deposition of Al can be shown by the equation.

$Al^{3+} + 3e^- \longrightarrow Al$

3 F 27 g

= 3 × 96500 C

This equation shows that : 3 × 96500 C deposit 27g of Al.

$48250\text{ C deposit} = \frac{27\times48350}{3\times96500}$

= 4.5 g of Al

42. (C) The existence of Fe^{2+} and NO^+ in nitropruside ion, $[Fe(CN)_5NO]^{2-}$ can be established by measuring the magnetic moment of the solid compound which should correspond to ($Fe^{2+} = 3d^6$) four unpaired electrons.

43. (B) Only the compound, $PhCHOHCH_3$ contains the grouping CH_3CHOH attached to C, therefore, it gives indoform test.

44. (C) Let the initial concentration of SO_2Cl_2

= 100 moles litre^{-1}

So, a = 100 moles litre^{-1}

a – x = (100 – x) moles litre^{-1}

$k = 2.2 \times 10^{-5}$ sec^{-1}

t = 90 × 60 sec = 5400 sec

$$2.2\times10^{-5} = \frac{2.303}{5400}\log\frac{100}{100-x}$$

$$\frac{100}{100-x} = \text{Antilog}\frac{5400\times2.2\times10^{-5}}{2.303}$$

$$\frac{100}{100-x} = 1.126$$

$$\therefore\ x = 100 - \frac{100}{1.126} = 11.2\%$$

45. (C) The complexes $CoCl_3 . 6\,NH_3$ and $PtCl_4 . 5\,NH_3$ are represented as $[Co(NH_3)_6]\,Cl_3$ and $[Pt(NH_3)_5Cl]Cl_3$ respectively.

$$[Co(NH_3)_6]Cl_3 \xrightarrow{\text{in solution}} [Co(NH_3)_6]^{3+} + 3Cl^-$$

$$[Pt(NH_3)_5Cl]Cl_3 \xrightarrow{\text{in solution}} [Pt(NH_3)_5Cl]^{3+} + 3Cl^-$$

As the number of ionic species in both the complexes is the same, their equimolar solutions will show approx. same conductance.

46. (D) The depression in the freezing point is given by, (subscript 1 is for solvent water and 2 is for solute, methyl alcohol).

$$\Delta T_f = \frac{1000\,g/kg\times K_f\times w_2}{w_1\times M_2}$$

$K_f = 1.86$ K kg mol^{-1}

Volume of water = 8 L = 8000 mL

So, Mass of water, w_1 = 8000 mL × 1 g/mL = 8000 g

Volume of alcohol = 2 L = 2000 mL

So, Mass of alcohol, w_2 = 2000 mL × 0.8 g/mL = 1600 g

Molar mass of methyl alcohol

M_2 = 32 g/mol

So,

$$\Delta T_f = \frac{1000\ g/kg \times 1.86\ K\ kg\ mol^{-1} \times 1600\ g}{8000\ g \times 32\ g\ mol^{-1}}$$

= 11.6 K = 11.6° C.

Therefore, Freezing point of the solution = 0°C – 11.6°C = – 11.6°C

So, the motor vehicle can be parked outdoors safely upto = – 11.6°C.

47. (Delete)

48. (B) $2NO_2 \underset{k_2}{\overset{k_1}{\rightleftharpoons}} N_2O_4$

$$\text{Rate} = -\frac{1}{2}\frac{d[NO_2]}{dt}$$

$= k_1\ [NO_2]^2 - k_2\ [N_2O_4]$

∴ Rate of disappearance of NO_2

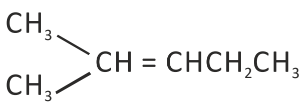
i.e., $$-\frac{d[NO_2]}{dt} = 2k_1[NO_2]^2 - 2k_2\ [N_2O_4]$$

49. (B) Out of CH_3COCH_3, CH_3CHO, CH_3CH_2CHO and $(CH_3)_2\ CHCHO$, two pairs containing a total of six carbon atoms are :

(i) CH_3COCH_3 + CH_3CH_2CHO

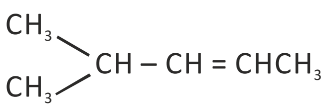
(ii) CH_3CHO + $(CH_3)_2CHCHO$

The alkenes which will give these pairs of compounds on ozonolysis are :



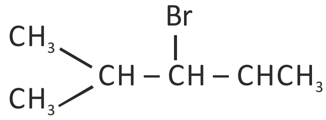
2 – Methyl – 2 – pentene (I)

and



4 – Methyl – 2 – pentene (II)

These two alkenes can be obtained as dehydrogenation products if the alkyl halide is



3 – Bromo – 2 – methylpentene

50. (B) Greater the lowering of V.P., greater is the depression in F. pt, i.e., lower is actual F pt.

51. (B) Mn^{2+} in $MnSO_4 . 4\ H_2O$ has d^5 configuration (five unpaired electrons);

Cu^{2+} in $CuSO_4 . 5\ H_2O$ has d^9 configuration (one upaired electron);

Fe^{2+} in $FeSO_4 . 6\ H_2O$ has d^6 configuration (four unpaired electrons);

Ni^{2+} in $NiSO_4 . 6\ H_2O$ has d^8 configuration (two unpaired electrons).

Thus, $CuSO_4 . 5\ H_2O$ has lowest degree of paramagnetism.

52. (D) Addition of Br_2 (trans) to trans-2-butene gives meso-2, 3-dibromobutane.

53. (A) Given values are reduction potentials $M^{+2} \rightarrow M^{3+} + e^-$ means oxidation. Oxidation potential of Cr will be highest and hence most easily oxidized.

54. (A) In metal carbonyls, the total bonding is M = C = 0. Thus, the bond order of C–O bond is reduced from triple bond to double bond. As a result, C–O bond length of 1.28 Å in CO increases to about 1.15 Å in many carbonyls.

55. (A) As NH group is electron-donating and o, p-directing while C = O is electron-withdrawing and m-directing, therefore, bromination will occur in the ring attached to the NH group predominantly at the unhindered p-position.

CRITICAL THINKING

56. (C) The passage mentions differences expected in 'executive functions' of the brain between children who have command of a single language and children who have mastered more than one.

However, it cannot be inferred that this effect continues as the number of languages continues to grow. For instance, it is not clear whether the difference in executive functions is also present between children who have command of two languages and children who have command of more than two languages.

57. (D)

Person	Department			Color						
	CISF	BSF	CRPF	Green	Blue	Red	Pink	Black	Violet	Purple
A	✓	✗	✗	✗	✗	✗	✗	✗	✓	✗
B	✗	✓	✗	✓	✗	✗	✗	✗	✗	✗
C	✗	✗	✓	✗	✓	✗	✗	✗	✗	✗
D	✓	✗	✗	✗	✗	✓	✗	✗	✗	✗
E	✗	✓	✗	✗	✗	✗	✗	✗	✗	✓
F	✓	✗	✗	✗	✗	✗	✗	✓	✗	✗
G	✗	✗	✓	✗	✗	✗	✓	✗	✗	✗

Person	Department	Colour
A	CISF	Violet
B	BSF	Green
C	CRPF	Blue
D	CISF	Red
E	BSF	Purple
F	CISF	Black
G	CRPF	Pink

Following the common explanation we get combination of D – CISF – Red is true.

58. (C) We are tempted to assume that technological progress is real progress. (1st line)

59. (B) Lets simplify the statements

Rohan : X will not go up unless Y goes down

Mahesh : When Y went Down X didn't change.

Mahesh thinks Rohan is saying when Y goes down X will go up. Find the answer which says this.

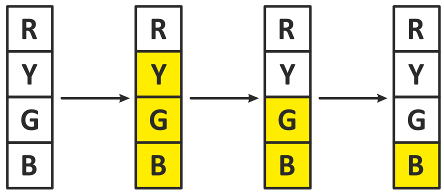
(A) housing prices will rise only if interest rates fall X will go up when Y goes down (Wrong)

(B) if interest rates fall, housing prices must rise when Y goes down X must rise (Correct)

(C) interest rates and housing prices tend to rise and fall together X and Y rise together and fall together(Wrong)

(D) interest rates are the only significant economic factor affecting housing prices Only Y affects X (not even close)

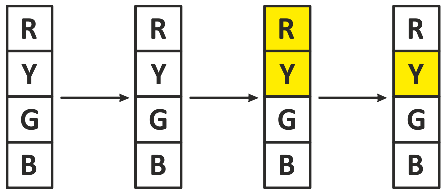
60. (C) If Switch A is fault :



. But given output is

not matched. Hence switch (A) is working.

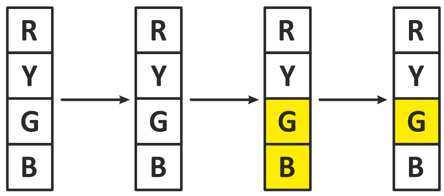
If Switch B is fault :



given output is

. Both are not matching. Hence switch (B) is working.

If Switch C is fault :



given output is

. Both are matched.

Hence switch (C) is fault.

Switch (C) is not given in the question figure.